These are outline solutions only. Each of the following should be fully explained for full marks.

1. (a) Describe each of the following lighting models:

(i) specular reflection;
(ii) diffuse reflection;
(iii) ambient lighting.

[9]

*(i) use cosine of the angle between normal and direction of perfect reflection (scalar product of the vectors);*
*(ii) use cosine of the angle between surface and incoming light*
*(iii) add a constant term to approximate light scattered within the environment.*

(b) Show how each of:

(i) constant shading;
(ii) Gouraud shading;
(iii) Phong shading

is computed, briefly stating the strength of each.

[6]

*(i) Each triangle given a constant colour. Fast.*
*(ii) Each vertex given a colour (by averaging adjacent normals to get a surface direction; then apply lighting model). Bi-linear interpolate across the triangle. Gives illusion of curved surfaces, hiding the edges.*
*(iii) Similar to Gouraud but bi-linear interpolate normals. apply lighting at every pixel. Highlights possible.*

What does radiosity attempt to achieve, how is this fundamentally different to the earlier methods and what are the practical constraints on how accurate it is?

[5]

*Radiosity seeks to solve the equations of light -scattering within the 3D model. Earlier methods do not consider these interactions at all - the shading is not modelled but applied by the renderer. Radiosity solves the equations independently of viewpoint and of screen resolution. Practical constraints are the accuracy of the surface subdivision; number of surface-to-surface interactions; accuracy of integration of the multiple components reaching a surface.*

2. (a) Describe the principles of Constructive Solid Geometry (CSG), when used as a method of modelling. Describe a suitable data structure for use in a computer-based solid modeller.

[6]

*CSG modelling uses a small number of primitives for which it can be computed: where a ray intersects the primitve; what the normal is at that point. These primitives define solids in space and are combined using logical operations. A binary tree is the obvious data structure. Each leaf is a primitive. Each internal node is a Boolean operation. The hierarchy permits complex shapes to be built, with a transform at each branch.*

(b) Describe the basis of the ray-tracing method of rendering. Explain how this method can be readily extended to include reflection, refraction and mist effects.

[6]

*Cast a ray (line) from the centre of projection through the screen pixel to be coloured. Calculate where the ray first intersects the model. Calculate the surface normal at this point and hence apply the lighting to determine the pixel's colour. Step to next pixel and repeat for entire picture. Reflection: allow the ray to continue at the mirror angle. Refraction: use Snell's law to change the direction of the ray as it goes between different optical densities. Mist: calculate the length of the ray in air and dilute the basic colour with white (desaturation) using an exponential relationship.*

(c) Carefully explain in detail how a solid model, represented by the data structure of part (a), can be ray-traced.

[8]

*[As given in the lecture notes but needs a careful, detailed explanation] The essence is: intersect ray with each leaf primitive to give an in-out segment; combine segments by working up the tree; have no reduced a 3D problem to a 1D problem; take nearest remaining intersection to viewer.*

3. (a) Explain the tri-stimulus theory of colour vision, with reference to the structure of the human retina.

[6]

*Three types of cones, with sensitivities peaking roughly in red, green and blue but spreading across most of the visible frequencies. Brain combines these so we see a colour image. Cones reside on the inner surface of the eye, where the lens forms the image.*

(b) Sketch the CIE diagram and use it to explain the following:

(i) no colour monitor can display all the colours we can see;

(ii) there are visible colours which are neither in the spectrum nor desaturated spectral colours.

[4]

*Draw the CIE diagram, marking colours such as red, green, blue (or indicate where the spectrum lies) and white.*

*(i) Sketch the CIE, with an interior triangle showing the monitor gamut. Show/argue that some visible (CIE) colours are outside the triangle, no matter how big it is, because the phosphors all lie within it and so therefor do linear combinations.*

*(ii) The purple line and all colours in the triangle bounded by it and white are visible but not on the spectrum or in the desaturated spectrum.*

(c) Use a similar diagram to show that linear interpolation in the CIE space differs in result from linear interpolation in RGB space.

[5]

*Several possible answers here. One example: in CIE, linear interpolation between pure red and pure green passes through desaturated colours (by inspection of the diagram). In RGB, linear interpolation between pure red (1,0,0) and pure green (0,1,0) remains fully saturated because the blue component must remain zero. Another example: in CIE, white is part way between red and deep blue; in RGB, white can never be achieved by interpolation because it is at one corner of the cube.*

(d) Why is it necessary to include negative values for RGB tri-stimulus coordinates and what is the basis for this in the human visual system?

[5]

*Explain that the eye's three kinds of colour receptors have a response which overlaps, so they are not independent. Describe the colour-matching experiment. Explain why this may not produce a match, that this can be fixed by adding a colour bias to the sample, that this is equivalent to taking it away from the resulting coordinates and that this will therefore give a negative result.*

4. (a) With the aid of a clear diagram, derive the formula for a rotation of theta about an arbitrary axis through the origin. You may assume that a rotation about the x axis is Rx, about the y axis is Ry and about the z axis is Rz but you should clearly show how you derive the cosine and sine terms needed. [There is no need to reproduce these matrices in full.]

[10]

*Bookwork. A good, large diagram is essential to the explanation. Explain the rotation order; how to get the cosine/sine terms etc.*

(b) What are homogeneous coordinates and what is their practical relevance to computer graphics?

[4]

*Embed n space in $(n+1)$ space: in this case embed 3D in 4D, adding a w coordinate. In practical terms, the matrices gain a row and a column. This allows us to combine additive operations (translation) with multiplicative ones (scale, rotations) in the same matrix.*

(c) Relate homogeneous coordinates to projective geometry. With the aid of a diagram, explain the perspective mapping of a one-dimensional space in homogeneous space.

[6]

*The 'extra' dimension allows us to project. In effect we can create a projected model and then collapse it onto the viewplane. Derivation requires a diagram, as in the 1D part of the notes.*