

CM10196: Discrete Mathematics for
Computation
Problem Sheet 5

Set November 9th 2007; hand in by Nov 22nd 2007

Coursework forms 25% of the assessment for this unit. Coursework will consist of your answers to eight problem sheets, plus the “learning log” exercise. Each problem sheet will be marked out of 10, and there will be 20 marks for the learning log.

On this sheet, each question has a different mark-weighting: see individual questions for details.

In the lectures we discussed the *structural* version of the natural numbers, where every natural number is represented as a term of the form

$$S(S(S(\dots(S(0))))).$$

The number of occurrences of S signifies the number being represented. This sheet asks you to define the operation of multiplication on these numbers, and prove some facts about it. You may freely use $+$ operation defined in the lectures, as well as the properties of $+$ which we proved in lectures.

1. Prove that the definition of $+$ we gave in lectures is associative, that is, that $a + (b + c) = (a + b) + c$ for all natural numbers a , b and c . (Hint: use induction on c .) [4 marks]
2. Give an inductive definition of the \times operation. [2 marks]
3. Prove that for any natural number n , $n \times S(0) = n$. [1 mark]
4. Prove that for any natural numbers m and n , $m \times n = n \times m$. (Hint: look at the proof of commutativity of $+$ in the lectures; notice how it is broken into lemmas; use the same approach.) [3 marks]

Remark Part (4) is considerably longer than the rest, but not worth so many marks. This is to encourage you to focus on getting the other questions right, but still to have a go at (4).