CM10196: Discrete Mathematics for Computation
Problem Sheet 4

Set November 2nd 2007; hand in by Nov 15th 2007

Coursework forms 25% of the assessment for this unit. Coursework will consist of your answers to eight problem sheets, plus the "learning log" exercise. Each problem sheet will be marked out of 10, and there will be 20 marks for the learning log.

On this sheet, each question is worth five marks.

1. In the lectures we discussed a technique for proving, by induction, statements of the form
$$\forall n \in \mathbb{N}. n \geq 2 \rightarrow P(n).$$
Such statements can be proved by establishing:

   Basis $P(2)$ holds.

   Inductive step If $P(n)$ holds then so does $P(n+1)$ for any $n \geq 2$.

   (a) Give a formula, along the lines of the induction principle formula from lectures, which makes this proof principle precise. [2 marks]

   (b) Show that your formula is valid, by showing how it follows from the usual "starting from zero" induction principle. There's a hint in the lecture notes. [3 marks]

2. Use induction to prove that every natural number $n \geq 2$ can be written as a product of prime numbers. That is, for any $n \geq 2$ there is a a collection of primes $p_1, \ldots p_k$ such that
$$n = p_1 \times \cdots \times p_k.$$
[5 marks: one for setting up the induction, one for the base case, and three for the inductive step.]