CM10196: Discrete Mathematics for Computation
Problem Sheet 3

Set October 22nd 2007; hand-in date set by your tutor

Coursework forms 25% of the assessment for this unit. Coursework will consist of your answers to eight problem sheets, plus the "learning log" exercise. Each problem sheet will be marked out of 10, and there will be 20 marks for the learning log.

On this sheet, each question is worth two marks.

1. Suppose a, b and c are three different elements. Let $A = \{a\}$, $B = \{b\}$, $C = \{c\}$, $D = \{a, b\}$, $E = \{a, c\}$, $F = \{b, c\}$ and $G = \{a, b, c\}$.

   Each expression below is equal to one of the sets A, B, $\ldots$, G, or the empty set $\emptyset$. Calculate the set described by each expression.

   (a) $A \cup B$.

   (b) $A \cup D$.

   (c) $D \cup E \cup F$.

   (d) $A \cap B$.

   (e) $G \cap F$.

   (f) $G \setminus F$.

   (g) $F \setminus G$.

   (h) $G \setminus (G \setminus F)$.

2. Two sets X and Y are called *disjoint* if $X \cap Y = \emptyset$.

   Among the sets A, $\ldots$, G defined above, there are three sets which have the property that no two are disjoint, but the intersection of all three is empty.

   (a) Find three such sets and show that they have this property.

   (b) Is your choice of three sets the only solution? If so, explain why. If not, give another solution.

3. Prove that for any three sets A, B and C,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Your proof should work by logical analysis of the definitions of union and intersection, rather than Venn diagrams.

4. Prove that for any sets A and B,

$$A \setminus (A \setminus B) = A \cap B.$$

Again, conduct your proof by means of logic rather than Venn diagrams.

5. Given a finite set X, the *cardinality* of X, written $|X|$, is the number of elements X has.

   (a) If $|X| = |Y| = 10$, what could $|X \cup Y|$ be? (The answer is a *range* of numbers.)

   (b) Give an example of sets X and Y such that $|X \cup Y|$ is as large as possible, and another example which makes $|X \cup Y|$ as small as possible.

   (c) What is the range of values of $|X \cap Y|$ when $|X| = |Y| = 10$?

   (d) Again, give examples which make this value as large as possible and as small as possible.