CM10196: Discrete Mathematics for Computation
Problem Sheet 2

Set October 15th 2007; hand-in date set by your tutor

Coursework forms 25% of the assessment for this unit. Coursework will consist of your answers to eight problem sheets, plus the "learning log" exercise. Each problem sheet will be marked out of 10, and there will be 20 marks for the learning log.

On this sheet, each question is worth two marks.

1. Write logical formulae to express the following statements.
   (a) Everyone is either male or female.
   (b) Every student who gets 40% or more on this course passes this course.
   (c) Not every student who passes gets 40%. (Use negation to express this.)
   (d) Not every student who passes gets 40%. (Do not use negation this time.)
   (e) Every student who understands logic can answer this question.

2. (a) Write this statement as a logical formula.

   > Every integer greater than one is either a prime number or divisible by a prime number.

   (An integer n is divisible by an integer m if n/m is itself an integer.)

   (b) Write the negation of the statement as a logical formula, without using any formulae of the form $\neg\forall x \ldots$ or $\neg\exists x \ldots$.

3. Suppose $P(x, y)$ is a predicate on pairs of integers, such that

$$\exists x \in \mathbb{Z}. \forall y \in \mathbb{Z}. P(x, y)$$

   is true. Which of the following are true? If you think a statement is true, explain why it is true; if you think it is false, give a counterexample.

(a) $\exists x \in \mathbb{Z}.\exists y \in \mathbb{Z}.P(x, y)$
(b) $\forall x \in \mathbb{Z}.\exists y \in \mathbb{Z}.P(x, y)$
(c) $\forall x \in \mathbb{Z}.\forall y \in \mathbb{Z}.P(x, y)$
(d) $\forall y \in \mathbb{Z}.\exists x \in \mathbb{Z}.P(x, y)$

4. Which of the following statements are equivalent to

$$\neg(\forall x \in \mathbb{Z}.\exists y \in \mathbb{Z}.P(x, y))?$$

Explain your answer.

(a) $\exists x \in \mathbb{Z}.\neg(\exists y \in \mathbb{Z}.P(x, y))$
(b) $\forall x \in \mathbb{Z}.\neg(\exists y \in \mathbb{Z}.P(x, y))$
(c) $\exists x \in \mathbb{Z}.\forall y \in \mathbb{Z}.\neg(P(x, y))$
(d) $\forall x \in \mathbb{Z}.\forall y \in \mathbb{Z}.\neg(P(x, y))$

5. (a) Write the statement "There is no largest prime number" as a logial formula.

(b) In the lectures, we gave a proof by contradiction that there are infinitely many prime numbers. This is essentially the same thing as proving that there is no largest prime number. Using your answer to question (5a), and the logical law of *reductio ad absurdum*, write a formula which expresses the mathematical argument used in this proof.