

# CM10196: Discrete Mathematics for Computation Problem Sheet 1

Set October 4th 2007; hand-in date set by your tutor

Coursework forms 25% of the assessment for this unit. Coursework will consist of your answers to eight problem sheets, plus the “learning log” exercise which will be specified in future weeks. Each problem sheet will be marked out of 10, and there will be 20 marks for the learning log.

On this sheet, each question is worth two marks.

1. Write a truth table for each of the following Boolean formulae.

(a)  $X \wedge (\neg X \rightarrow (X \vee \neg Y))$

(b)  $(X \rightarrow Y) \rightarrow ((X \rightarrow (Y \rightarrow Z)) \rightarrow (X \rightarrow Z))$

(c)  $(X \wedge (X \rightarrow Y)) \rightarrow Y$

(d)  $(X \rightarrow Y) \rightarrow (Y \rightarrow X)$

Which of them are tautologies?

2. The *contrapositive* of an implication  $X \rightarrow Y$  is the implication  $\neg Y \rightarrow \neg X$ . These two formulae are logically equivalent. Show this using truth tables. (For extra mental fun, convince yourself that it is reasonable!)
3. The regulations of the university state that: if a student achieves 40% in a course then the student should pass that course. Mr Notquite achieved 39%. The board of examiners would like to award him a pass anyway. A member of staff who is not very good at logic thinks that the regulations prevent this.

Demonstrate that the member of staff is wrong by formulating a Boolean formula which would say that Mr Notquite cannot pass, and showing that it is not a tautology.

4. (a) Draw up a truth table for the NAND operator described in lectures.

- (b) Show how to express  $\neg$ ,  $\wedge$ ,  $\vee$  and  $\rightarrow$  using just NAND. Prove that you are right either by means of truth tables or by algebraic reasoning, or a combination.
5. Express the following formulae in disjunctive normal form and in conjunctive normal form. Show your reasoning.
- (a)  $(X \rightarrow Y) \wedge (Y \rightarrow X)$ .
- (b)  $\neg(X \wedge Y \wedge \neg Z) \rightarrow (\neg X \vee Y)$