Exercise 7.1
Consider the following program $P$:

$$
p(X,a).
p(X,f(Y)) :- p(X,Y), q(X,Y).
q(X,Y) :- r(Y).
q(f(X),Y) :- q(X,f(Y)).
\]

Show that the query $? - p(f(f(a)), q(f(X), f(Y)))$ is $n$-deep with respect to $P$, for $n = 6, 7, 8, 9$ (you may find the notion of $n$-depth in the 4th series of slides, page 28.)

Exercise 7.2
Consider the following program $P$:

$$
ext except([],X,[]).
ext except([X|Xs],Ys,Z) :- member(X,Ys), except(Xs,Ys,Z).
ext except([X|Xs],Ys,[X|Z]) :- except(Xs,Ys,Z).
\]

This specific implementation of except/3 is supposed to be used only in modality except(Input, Input, Output), with the two inputs ground. We want the output to be the result of eliminating items from the first argument that occur in the second one. For example, we would like that the program computes the following:

?- except([a,b,c],[b,b],Z).
Z = [a, c].

Yet $P$ produces the following result:

?- except([a,b,c],[b,b],Z).
Z = [a, c] ;
Z = [a, c] ;
Z = [a, b, c].
Two problems are evident, that we need to fix. First, it returns the correct answer more than once (in this case, twice); second, it returns some wrong answer(s) (in this case, the last one).

(a) Show how to solve both problems, simultaneously, using cut.

(b) Interrogate $P$ with the following query:

$$?- \text{except}([a,b,c],[b,b],Z),!.$$  

explain what you observe and compute.

Exercise 7.3

Consider the following program $P$, where clauses have been numbered:

1. even(0).
2. even(s(s(X))) :- even(X).
3. num(X) :- even(X).
4. num(X) :- ~even(X).

(a) Draw the SLDNF-tree for the $P \cup \{Q\}$ where $Q$ is $?- \neg \text{num}(s(s(s(0))))$.

(b) The clause 3 is now modified by $3' \text{num}(X) :- \neg \text{even}(X),!$. The program so modified is called $P'$. Explain the effects of this modification in the computation of $P' \cup \{Q\}$.

(c) Explain what would happen if in $P'$ you swap the order of clauses $3'$ and 4, for the same query.

(d) Is the query $Q$ allowed? Is the given program $P$ allowed?