Exercise 3.1
For each of the following specify the type (query, clause, fact):

(a) \( \neg p(X,Y) \).
(b) \( p(X,Y) : \neg q(X) \).
(c) \( p(X,Y) : \neg r \).
(d) \( p(X,Y), q(X,Y) : \neg s(X,Y) \).
(e) \( \neg p(X,Y), s(X,Y) \).

Hint: there might be one which is of none of the kinds or is of more than one kind.

Exercise 3.2
Consider the following program and queries:

Program:

\[ p(X,Y) : \neg q(X,Y), r(Y,X) \].
\[ q(X,a) : \neg s(X) \].
\[ q(X,c) : \neg s(X) \].
\[ r(X,b) : \neg t(X) \].
\[ s(a) \].
\[ s(b) \].
\[ s(c) \].
\[ t(a) \].
\[ t(c) \].

Queries:

(I) \( q(a,b) \).
(II) \( q(c,a) \).
(III) \( p(X,c) \).
(IV) \( p(a,X) \).
(V) \( p(X,Y) \).

(a) Give a selection and a computation rule.
(b) For each query give the SLD-derivation and for each step give the selected atom (use the selection rules that you have provided in the previous part), the clause (standardize the names apart) used and the resultant. Follow the SLD resolution until it finitely succeeds or fails.
Exercise 3.3
Give a program $P$ and a query $Q$ such that for some substitution $\theta$ there is a proof of $Q\theta$ (i.e. there is a successful SLD-derivation providing $\theta$) via some selection rule $R$ which cannot be found if Standardization Apart (Slide 3/15) were not required.

Exercise 3.4
(a) Give a program $P$, a query $Q$ and two selection rules $R_1$ and $R_2$ such that:
   - every SLD-derivation of $P \cup \{Q\}$ via $R_1$ is infinite
   - every SLD-derivation of $P \cup \{Q\}$ via $R_2$ is failed
(b) Is it possible to construct $P$ and $Q$ such that additionally to the properties specified above there exists a successful SLD-derivation via some selection rule $R_3$? Justify your answer.

Exercise 3.5
Consider the following program:

\[
p(X) :- q(X), r(X).
q(f(X)).
r(f(a)).
\]

(a) Give an SLD-derivation $\xi$ for query $?- p(X)$ that uses the Prolog selection and computation rules (See the end of this exercise list).
(b) For each derivation step of $\xi$, give the resultant that is associated with this step (Sl. 3/18).
(c) Give the resultants of every level $i$ of $\xi$ (Sl. 3/19).

Exercise 3.6
Consider the query $?- double(s(0),Y), double(Y,s(s(0)))$. together with the program

\[
double(0,0).
double(s(N),s(s(F))):- double(N,F).
\]

(a) Give an SLD-derivation using the Prolog selection and computation rules (See the end of this exercise list). Give the substitutions and the CAS(computed answer substitution).
(b) Show that the Switching Lemma (Slide 3/26) holds for the initial query (i.e., for $n = 0$).
   Hint: Give a second SLD-derivation selecting the second atom at the beginning and using the Prolog selection and computation rules (See the end of this exercise list) afterwards. Show the correspondence of both derivations.

Exercise 3.7
Give the SLD-tree for query $V$ of Exercise 3.2(use Prolog’s selection and computation rules).

Prolog’s selection rule: Prolog always selects the left most atom of the query.
Prolog’s computation rule: Prolog always selects clauses in a top down manner.