Exercise 10.1
Consider the following program $P$:

$$
\text{leq}(0, X),
\text{leq}(s(X), s(Y)) : \neg \text{leq}(X, Y).
$$

$$
\text{min}(U, V, U) : \neg \text{leq}(U, V).
\text{min}(U, V, V) : \neg \text{leq}(V, U).
$$

Consider the following level mapping $\| \cdot \|$ defined on the predicates of $P$,

$$
\| \text{leq}(t_1, t_2) \| = |t_1|,
|t_1\ =
|t_2| = 0,
\| \text{min}(t_1, t_2, t_3) \| = |t_1| + |t_2| + 1
$$

where, on terms, $|0| = 0$ and $|s(n)| = |n| + 1$. Moreover, consider the following interpretation:

$$
I = \{ \text{min}(a, b, b) \mid a, b \text{ are ground} \} \cup \{ \text{min}(a, b, a) \mid a, b \text{ are ground} \} \cup
\{ \text{leq}(a, b) \mid a, b \text{ are ground} \} \cup \{ a \neq b \mid a, b \text{ are ground} \}.
$$

(a) Is $P$ acceptable wrt the given $\| \cdot \|$ and $I$?

(b) Verify (showing your details) if the following queries are bounded wrt $\| \cdot \|$ and $I$:

$$
\| \text{min}(s(s(s(s(0)))), s(s(0)), Y), \text{min}(s(s(s(s(0)))), Y, s(s(0)))) \|,
\| \text{min}(s(s(s(s(0)))), Y, Y), \text{min}(s(s(s(0)))), Y, s(s(0))) \|.
$$

Exercise 10.2
Consider the following program $P$ implementing the Ackermann function:

$$
\text{ack}(0, Y, s(Y)),
\text{ack}(s(X), 0, Z) : \neg \text{ack}(X, s(0), Z),
\text{ack}(s(X), s(Y), Z) : \neg \text{ack}(s(X), Y, A), \text{ack}(X, A, Z).
$$

Consider the following interpretation $I$ and level mapping $\| \cdot \|$ on predicate $\text{ack}$ (returning pairs), which in turn is defined on terms:

$$
I = \{ \text{ack}(t_1, t_2, t_3) \mid t_1, t_2, t_3 \text{ are ground} \}
$$

$$
\| \text{ack}(n, m, a) \| = \langle |n|, |m| \rangle,
0 = 0,
|s(n)| = 1 + |n|
$$

Show that $P$ is acceptable wrt $I$ and $\| \cdot \|$.