Parse Tables

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Material taken from:

Robin Hunter
The Essence of Compilers
Prentice Hall

A grammar

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow x$

An SLR(1) parse table for the grammar

	input symbol								
	E	T	F	+	$*$	(	)	x	$\perp$
state 1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

Sn = shift to state n:

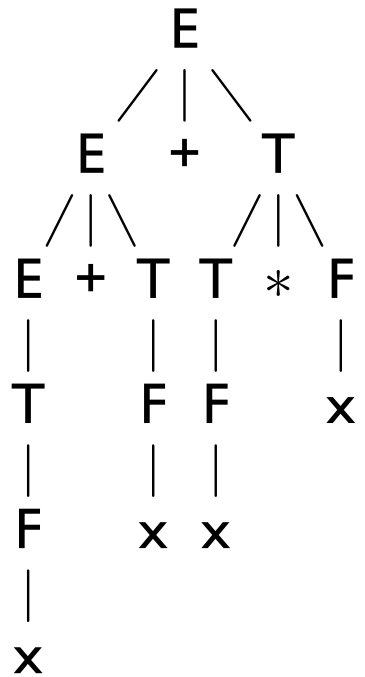
1. input symbol $\rightarrow$ symbol stack
2. next terminal $\rightarrow$ input symbol
3. new state $\rightarrow$ state stack

Rn = reduce with $n : A \rightarrow \beta$:

1. remove $|\beta|$ symbols from symbol stack
2. remove $|\beta|$ states from state stack
3. $A \rightarrow$ input symbol

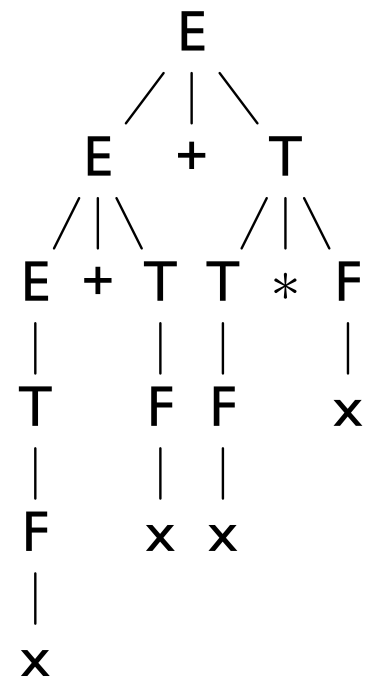
A grammar and a parse tree

1	$E \rightarrow E + T$
2	$E \rightarrow T$
3	$T \rightarrow T * F$
4	$T \rightarrow F$
5	$F \rightarrow (E)$
6	$F \rightarrow x$



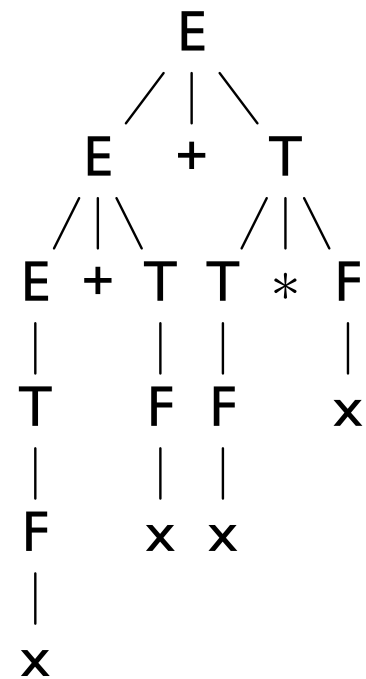
Bottom-up parsing based on the parse table

input string	input symbol	symbol stack	state stack	S/R
x+x+x∗x	x		1	S12
+x+x∗x	+	x	1 12	R6 (F → x)
+x+x∗x	F		1	S8
+x+x∗x	+	F	1 8	R4 (T → F)
+x+x∗x	T		1	S5
+x+x∗x	+	T	1 5	R2 (E → T)
+x+x∗x	E		1	S2
+x+x∗x	+	E	1 2	S3
x+x∗x	x	E+	1 2 3	S12
+x∗x	+	E+x	1 2 3 12	R6 (F → x)
+x∗x	F	E+	1 2 3	S8
+x∗x	+	E+F	1 2 3 8	R4 (T → F)
+x∗x	T	E+	1 2 3	S4
+x∗x	+	E+T	1 2 3 4	R1 (E → E+T)
+x∗x	E		1	S2
+x∗x	+	E	1 2	S3
x∗x	x	E+	1 2 3	S12
∗x	∗	E+x	1 2 3 12	R6 (F → x)
∗x	F	E+	1 2 3	S8
∗x	∗	E+F	1 2 3 8	R4 (T → F)
∗x	T	E+	1 2 3	S4
∗x	∗	E+T	1 2 3 4	S6
x	x	E+T∗	1 2 3 4 6	S12
	⊥	E+T∗x	1 2 3 4 6 12	R6 (F → x)
	F	E+T∗	1 2 3 4 6	S7
	⊥	E+T∗F	1 2 3 4 6 7	R3 (T → T∗F)
	T	E+	1 2 3	S4
	⊥	E+T	1 2 3 4	R1 (E → E+T)
	E		1	S2



Bottom-up parsing based on the parse table

input string	input symbol	symbol stack	state stack	S/R
x+x+x∗x	x		1	S12
+x+x∗x	+	x	1 12	R6 (F → x)
+x+x∗x	F		1	S8
+x+x∗x	+	F	1 8	R4 (T → F)
+x+x∗x	T		1	S5
+x+x∗x	+	T	1 5	R2 (E → T)
+x+x∗x	E		1	S2
+x+x∗x	+	E	1 2	S3
x+x∗x	x	E+	1 2 3	S12
+x∗x	+	E+x	1 2 3 12	R6 (F → x)
+x∗x	F	E+	1 2 3	S8
+x∗x	+	E+F	1 2 3 8	R4 (T → F)
+x∗x	T	E+	1 2 3	S4
+x∗x	+	E+T	1 2 3 4	R1 (E → E+T)
+x∗x	E		1	S2
+x∗x	+	E	1 2	S3
x∗x	x	E+	1 2 3	S12
∗x	∗	E+x	1 2 3 12	R6 (F → x)
∗x	F	E+	1 2 3	S8
∗x	∗	E+F	1 2 3 8	R4 (T → F)
∗x	T	E+	1 2 3	S4
∗x	∗	E+T	1 2 3 4	S6
x	x	E+T∗	1 2 3 4 6	S12
	⊥	E+T∗x	1 2 3 4 6 12	R6 (F → x)
	F	E+T∗	1 2 3 4 6	S7
	⊥	E+T∗F	1 2 3 4 6 7	R3 (T → T∗F)
	T	E+	1 2 3	S4
	⊥	E+T	1 2 3 4	R1 (E → E+T)
	E		1	S2



Parse from the left — reduce from the right.

Parsing process — Step 1

1	$E \rightarrow E + T$
2	$E \rightarrow T$
3	$T \rightarrow T * F$
4	$T \rightarrow F$
5	$F \rightarrow (E)$
6	$F \rightarrow x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
x+x+x∗x	**x**		**1**	**S12**
+x+x∗x	+	x	1 12	R6 ($F \rightarrow x$)
+x+x∗x	F		1	S8

Parsing process — Step 2

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
x+x+x∗x	**x**		**1**	**S12**
+x+x∗x	**+**	**x**	1 **12**	**R6** ($F \to x$)
+x+x∗x	F		1	S8

Parsing process — Step 3

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
x+x+x∗x	x		1	S12
+x+x∗x	**+**	**x**	1 **12**	**R6** (F → x)
+x+x∗x	**F**		**1**	**S8**

Parsing process — Step 4

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x+x∗x	+	x	1 12	R6 ($F \to x$)
+x+x∗x	**F**		**1**	**S8**
+x+x∗x	**+**	**F**	1 **8**	**R4** ($T \to F$)

Parsing process — Step 5

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x+x∗x	F		1	S8
+x+x∗x	+	F	1 8	R4 ($T \to F$)
+x+x∗x	T		1	S5

Parsing process — Step 6

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x+x∗x	+	F	1 8	R4 ($T \to F$)
+x+x∗x	**T**		**1**	**S5**
+x+x∗x	**+**	**T**	1 **5**	**R2** ($E \to T$)

Parsing process — Step 7

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x+x∗x	T		1	S5
+x+x∗x	+	T	1 5	R2 ($E \to T$)
+x+x∗x	E		1	S2

Parsing process — Step 8

1	$E \rightarrow E + T$
2	$E \rightarrow T$
3	$T \rightarrow T * F$
4	$T \rightarrow F$
5	$F \rightarrow (E)$
6	$F \rightarrow x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x+x∗x	+	T	1 5	R2 ($E \rightarrow T$)
+x+x∗x	**E**		**1**	**S2**
+x+x∗x	**+**	**E**	1 **2**	**S3**

Parsing process — Step 9

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	*	(	)	x	⊥
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x+x∗x	E		1	S2
+x+x∗x	**+**	E	1 **2**	**S3**
x+x∗x	**x**	E**+**	1 2 **3**	**S12**

Parsing process — Step 10

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x+x∗x	+	E	1 2	S3
x+x∗x	**x**	E+	1 2 **3**	**S12**
+x∗x	**+**	E+**x**	1 2 3 **12**	**R6** ($F \to x$)

Parsing process — Step 11

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
x+x∗x	x	E+	1 2 3	S12
+x∗x	**+**	E+**x**	1 2 3 **12**	**R6** ($F \to x$)
+x∗x	**F**	E+	1 2 **3**	**S8**

Parsing process — Step 12

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x∗x	+	E+x	1 2 3 12	R6 ($F \to x$)
+x∗x	**F**	E+	1 2 **3**	**S8**
+x∗x	**+**	E+**F**	1 2 3 **8**	**R4** ($T \to F$)

Parsing process — Step 13

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x∗x	F	E+	1 2 3	S8
+x∗x	**+**	E+**F**	1 2 3 **8**	**R4** ($T \to F$)
+x∗x	**T**	E+	1 2 **3**	**S4**

Parsing process — Step 14

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x∗x	+	E+F	1 2 3 8	R4 ($T \to F$)
+x∗x	**T**	E+	1 2 **3**	**S4**
+x∗x	**+**	E+**T**	1 2 3 **4**	**R1** ($E \to E + T$)

Parsing process — Step 15

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x∗x	T	E+	1 2 3	S4
+x∗x	**+**	**E+T**	1 **2 3 4**	**R1** (E → E + T)
+x∗x	**E**		**1**	**S2**

Parsing process — Step 16

1	$E \rightarrow E + T$
2	$E \rightarrow T$
3	$T \rightarrow T * F$
4	$T \rightarrow F$
5	$F \rightarrow (E)$
6	$F \rightarrow x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x∗x	+	E+T	1 2 3 4	R1 ($E \rightarrow E + T$)
+x∗x	**E**		**1**	**S2**
+x∗x	**+**	**E**	1 **2**	**S3**

Parsing process — Step 17

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	*	(	)	x	⊥
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x∗x	E		1	S2
+x∗x	**+**	E	1 **2**	**S3**
x∗x	**x**	E**+**	1 2 **3**	**S12**

Parsing process — Step 18

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
+x∗x	+	E	1 2	S3
x∗x	**x**	E+	1 2 **3**	**S12**
∗x	∗	E+**x**	1 2 3 **12**	**R6** ($F \to x$)

Parsing process — Step 19

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
x$*$x	x	E+	1 2 3	S12
$*$x	$*$	E+**x**	1 2 3 **12**	**R6** ($F \to x$)
$*$x	**F**	E+	1 2 **3**	**S8**

Parsing process — Step 20

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
$*x$	$*$	E+x	1 2 3 12	R6 ($F \to x$)
$*x$	**F**	E+	1 2 **3**	**S8**
$*x$	$*$	E+**F**	1 2 3 **8**	**R4** ($T \to F$)

Parsing process — Step 21

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
$*x$	F	E+	1 2 3	S8
$*x$	$*$	E+**F**	1 2 3 **8**	**R4** ($T \to F$)
$*x$	**T**	E+	1 2 **3**	**S4**

Parsing process — Step 22

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
$*x$	$*$	E+F	1 2 3 8	R4 ($T \to F$)
$*x$	**T**	E+	1 2 **3**	**S4**
$*x$	$*$	E+**T**	1 2 3 **4**	**S6**

Parsing process — Step 23

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	*	(	)	x	⊥
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
$*x$	T	E+	1 2 3	S4
$*x$	$*$	E+T	1 2 3 **4**	**S6**
x	**x**	E+T$*$	1 2 3 4 **6**	**S12**

Parsing process — Step 24

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	*	(	)	x	⊥
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
*x	*	E+T	1 2 3 4	S6
x	**x**	E+T*	1 2 3 4 **6**	**S12**
	⊥	E+T***x**	1 2 3 4 6 **12**	**R6** (F → x)

Parsing process — Step 25

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
x	x	E+T$*$	1 2 3 4 6	S12
	$\perp$	E+T$*$**x**	1 2 3 4 6 **12**	**R6** ($F \to x$)
	F	E+T$*$	1 2 3 4 **6**	**S7**

Parsing process — Step 26

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
	$\perp$	E+T$*$x	1 2 3 4 6 12	R6 ($F \to x$)
	F	E+T$*$	1 2 3 4 **6**	**S7**
	$\perp$	E+T$*$**F**	1 2 3 4 6 **7**	**R3** ($T \to T * F$)

Parsing process — Step 27

1	$E \rightarrow E + T$
2	$E \rightarrow T$
3	$T \rightarrow T * F$
4	$T \rightarrow F$
5	$F \rightarrow (E)$
6	$F \rightarrow x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
	F	E+T$*$	1 2 3 4 6	S7
	$\perp$	E+**T$*$F**	1 2 3 **4 6 7**	**R3** ($T \rightarrow T * F$)
	T	E+	1 2 **3**	**S4**

Parsing process — Step 28

1	$E \to E + T$
2	$E \to T$
3	$T \to T * F$
4	$T \to F$
5	$F \to (E)$
6	$F \to x$

	E	T	F	+	$*$	(	)	x	$\perp$
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
	$\perp$	E+T$*$F	1 2 3 4 6 7	R3 ($T \to T * F$)
	T	E+	1 2 **3**	**S4**
	$\perp$	E+**T**	1 2 3 **4**	**R1** ($E \to E + T$)

Parsing process — Step 29

1	$E \rightarrow E + T$
2	$E \rightarrow T$
3	$T \rightarrow T * F$
4	$T \rightarrow F$
5	$F \rightarrow (E)$
6	$F \rightarrow x$

	E	T	F	+	*	(	)	x	⊥
1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7				R3	R3		R3		R3
8				R4	R4		R4		R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11				R5	R5		R5		R5
12				R6	R6		R6		R6

input string	input symbol	symbol stack	state stack	S/R
	T	E+	1 2 3	S4
	⊥	**E+T**	1 **2 3 4**	**R1** ($E \rightarrow E + T$)
	E		**1**	

Annotating the grammar with states

$$
\begin{aligned}
E &\rightarrow E + T \\
E &\rightarrow T \\
T &\rightarrow T * F \\
T &\rightarrow F \\
F &\rightarrow (E) \\
F &\rightarrow x
\end{aligned}
$$

Annotating the grammar with states

$$
\begin{aligned}
E &\to \ | \ E \ + \ T \\
E &\to \ | \ T \\
T &\to \ | \ T \ * \ F \\
T &\to \ | \ F \\
F &\to \ | \ (\ E \ ) \\
F &\to \ | \ x
\end{aligned}
$$

Annotating the grammar with states

$$
\begin{array}{lll}
E \to & | & E_2 \quad + \quad T \\
E \to & | & T \\
T \to & | & T \quad * \quad F \\
T \to & | & F \\
F \to & | & (\; E \;) \\
F \to & | & x
\end{array}
$$

Annotating the grammar with states

$$
\begin{aligned}
E &\rightarrow {}_{1}\ E_2 \quad +\ {}_{3}T \\
E &\rightarrow {}_{1}\ T \\
T &\rightarrow {}_{1,3}\ T \quad * \ F \\
T &\rightarrow {}_{1,3}\ F \\
F &\rightarrow {}_{1,3}\ (\ E\ ) \\
F &\rightarrow {}_{1,3}\ x
\end{aligned}
$$

Annotating the grammar with states

$$
\begin{aligned}
E &\rightarrow {}_{1}\ E_2 \quad +\ {}_{3}T_4 \\
E &\rightarrow {}_{1}\ T \\
T &\rightarrow {}_{1,3}\ T\ {}_{,4} * \ F \\
T &\rightarrow {}_{1,3}\ F \\
F &\rightarrow {}_{1,3}\ (\ E\ ) \\
F &\rightarrow {}_{1,3}\ x
\end{aligned}
$$

Annotating the grammar with states

$$
\begin{aligned}
E &\rightarrow {}_{1}\ E_{2} \quad +\ {}_{3}T_{4} \\
E &\rightarrow {}_{1}\ T_{5} \\
T &\rightarrow {}_{1,3}\ T_{5,4} * \ F \\
T &\rightarrow {}_{1,3}\ F \\
F &\rightarrow {}_{1,3}\ \ (\ E \ ) \\
F &\rightarrow {}_{1,3}\ \ x
\end{aligned}
$$

Annotating the grammar with states

$$
\begin{aligned}
E &\rightarrow {}_{1}\ E_{2} \quad +\ {}_{3}T_{4} \\
E &\rightarrow {}_{1}\ T_{5} \\
T &\rightarrow {}_{1,3}\ T_{5,4} * {}_{6}F \\
T &\rightarrow {}_{1,3}\ F \\
F &\rightarrow {}_{1,3,6}\ (\ E\ ) \\
F &\rightarrow {}_{1,3,6}\ x
\end{aligned}
$$

Annotating the grammar with states

$$
\begin{aligned}
E &\rightarrow {}_{1}\ E_{2} \quad + {}_{3} T_{4} \\
E &\rightarrow {}_{1}\ T_{5} \\
T &\rightarrow {}_{1,3}\ T_{5,4} * {}_{6} F_{7} \\
T &\rightarrow {}_{1,3}\ F \\
F &\rightarrow {}_{1,3,6}\ (\ E\ ) \\
F &\rightarrow {}_{1,3,6}\ x
\end{aligned}
$$

Annotating the grammar with states

$$
\begin{aligned}
E &\rightarrow {}_{1}\ E_{2} \quad + {}_{3} T_{4} \\
E &\rightarrow {}_{1}\ T_{5} \\
T &\rightarrow {}_{1,3}\ T_{5,4} * {}_{6} F_{7} \\
T &\rightarrow {}_{1,3}\ F_{8} \\
F &\rightarrow {}_{1,3,6}\ (\ E\ ) \\
F &\rightarrow {}_{1,3,6}\ x
\end{aligned}
$$

Annotating the grammar with states

$$
\begin{aligned}
E &\rightarrow {}_{1,9}E_{2} \quad + {}_{3}T_{4} \\
E &\rightarrow {}_{1,9}T_{5} \\
T &\rightarrow {}_{1,3,9}T_{5,4} * {}_{6}F_{7} \\
T &\rightarrow {}_{1,3,9}F_{8} \\
F &\rightarrow {}_{1,3,6,9}(\,{}_{9}E\,) \\
F &\rightarrow {}_{1,3,6,9}x
\end{aligned}
$$

Annotating the grammar with states

$$
\begin{aligned}
E &\rightarrow {}_{1,9}E_{2,10} + {}_{3}T_{4} \\
E &\rightarrow {}_{1,9}T_{5} \\
T &\rightarrow {}_{1,3,9}T_{5,4} * {}_{6}F_{7} \\
T &\rightarrow {}_{1,3,9}F_{8} \\
F &\rightarrow {}_{1,3,6,9}(_{9}E_{10}) \\
F &\rightarrow {}_{1,3,6,9}x
\end{aligned}
$$

Annotating the grammar with states

$$
\begin{aligned}
E &\rightarrow {}_{1,9}E_{2,10} + {}_{3}T_{4} \\
E &\rightarrow {}_{1,9}T_{5} \\
T &\rightarrow {}_{1,3,9}T_{5,4} * {}_{6}F_{7} \\
T &\rightarrow {}_{1,3,9}F_{8} \\
F &\rightarrow {}_{1,3,6,9}(_{9}E_{10})_{11} \\
F &\rightarrow {}_{1,3,6,9}x
\end{aligned}
$$

Annotating the grammar with states

$$
\begin{aligned}
E &\rightarrow {}_{1,9}E_{2,10} + {}_{3}T_{4} \\
E &\rightarrow {}_{1,9}T_{5} \\
T &\rightarrow {}_{1,3,9}T_{5,4} * {}_{6}F_{7} \\
T &\rightarrow {}_{1,3,9}F_{8} \\
F &\rightarrow {}_{1,3,6,9}(_{9}E_{10})_{11} \\
F &\rightarrow {}_{1,3,6,9}x_{12}
\end{aligned}
$$

Shift actions in the parse table

1 $E \rightarrow {}_{1,9}E_{2,10} + {}_{3}T_{4}$
2 $E \rightarrow {}_{1,9}T_{5}$
3 $T \rightarrow {}_{1,3,9}T_{5,4} * {}_{6}F_{7}$
4 $T \rightarrow {}_{1,3,9}F_{8}$
5 $F \rightarrow {}_{1,3,6,9}(_{9}E_{10})_{11}$
6 $F \rightarrow {}_{1,3,6,9}x_{12}$

	input symbol								
	E	T	F	+	*	(	)	x	⊥
state 1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4					S6				
5					S6				
6			S7			S9		S12	
7									
8									
9	S10	S5	S8			S9		S12	
10				S3			S11		
11									
12									

Conflict-less reduction actions in the parse table

1 $E \rightarrow {}_{1,9}E_{2,10} + {}_{3}T_{4}$
2 $E \rightarrow {}_{1,9}T_{5}$
3 $T \rightarrow {}_{1,3,9}T_{5,4} * {}_{6}F_{7}$
4 $T \rightarrow {}_{1,3,9}F_{8}$
5 $F \rightarrow {}_{1,3,6,9}(_{9}E_{10})_{11}$
6 $F \rightarrow {}_{1,3,6,9}x_{12}$

	input symbol								
	E	T	F	+	*	(	)	x	⊥
state 1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4					S6				
5					S6				
6			S7			S9		S12	
7	R3	R3	R3	R3	R3	R3	R3	R3	R3
8	R4	R4	R4	R4	R4	R4	R4	R4	R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11	R5	R5	R5	R5	R5	R5	R5	R5	R5
12	R6	R6	R6	R6	R6	R6	R6	R6	R6

Conflict-less reduction actions in the parse table

1 $E \rightarrow {}_{1,9}E_{2,10} + {}_{3}T_{4}$
2 $E \rightarrow {}_{1,9}T_{5}$
3 $T \rightarrow {}_{1,3,9}T_{5,4} * {}_{6}F_{7}$
4 $T \rightarrow {}_{1,3,9}F_{8}$
5 $F \rightarrow {}_{1,3,6,9}(_{9}E_{10})_{11}$
6 $F \rightarrow {}_{1,3,6,9}x_{12}$

	input symbol								
	E	T	F	+	$*$	(	)	x	$\perp$
state 1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4					S6				
5					S6				
6			S7			S9		S12	
7	R3	R3	R3	R3	R3	R3	R3	R3	R3
8	R4	R4	R4	R4	R4	R4	R4	R4	R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11	R5	R5	R5	R5	R5	R5	R5	R5	R5
12	R6	R6	R6	R6	R6	R6	R6	R6	R6

What about states 4 and 5 (reducing with 1 and 2)?

We have shift/reduce conflicts in correspondence with $*$.

Solution of the shift/reduce conflicts

1 $E \rightarrow {}_{1,9}E_{2,10} + {}_{3}T_{4}$
2 $E \rightarrow {}_{1,9}T_{5}$
3 $T \rightarrow {}_{1,3,9}T_{5,4} * {}_{6}F_{7}$
4 $T \rightarrow {}_{1,3,9}F_{8}$
5 $F \rightarrow {}_{1,3,6,9}(_{9}E_{10})_{11}$
6 $F \rightarrow {}_{1,3,6,9}x_{12}$

	input symbol								
	E	T	F	+	*	(	)	x	⊥
state 1	S2	S5	S8			S9		S12	
2				S3					
3		S4	S8			S9		S12	
4				R1	S6		R1		R1
5				R2	S6		R2		R2
6			S7			S9		S12	
7	R3	R3	R3	R3	R3	R3	R3	R3	R3
8	R4	R4	R4	R4	R4	R4	R4	R4	R4
9	S10	S5	S8			S9		S12	
10				S3			S11		
11	R5	R5	R5	R5	R5	R5	R5	R5	R5
12	R6	R6	R6	R6	R6	R6	R6	R6	R6

We look for the followers of E. Check Steps 7, 15 and 29.

No further conflicts: LR(1) table (can be improved to SLR(1)).